# THE CRITERIA OF REALITY IN THE PERIODOGRAM



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In a recent paper in the Quarterly Journal of the Royal Meteorological Society (*1*), Sir Gilbert T. Walker has developed equations for criteria of reality of periodicities, and has discussed the periodogram and other methods of examining data for them. In this article he refers to an older paper of his, published in the Indian Meteorological Memoirs, (*2*). I had seen neither of these papers when I wrote a discussion of the method in connection with an application to rainfall data of the world (*3*).

Some of the points I have discussed have been treated by Walker in these papers and in a more elegant fashion than mine. His papers are perhaps the most valuable contributions to the theory since Schuster published his original developments (*4*).

However, although Walker's mathematics is entirely correct, he makes an apparent error of application of the equation he has derived. Before discussing this it may not be out of place to review his development of a criterion for reality of a periodogram peak, since the Indian Meteorological Memoirs are not available to a rather large number of meteorologists. The exact form of development and the notation here are different from his, although the results are the same.

Suppose that a periodogram has been computed. If $I_m$ is the mean intensity and $I$ the intensity of any peak:

$$\text{Define } H \equiv \frac{I}{I_m}$$

Schuster shows that the probability, or more properly named the expectancy ratio, of any one specified peak being of height $h$, assuming merely accidental variations of the data, is $e^{-h}$. As Walker points out, it is obvious that in our periodogram, with its many peaks from which to choose, we are much more likely to find one of this height.

Since the expectancy of any point being this high or higher is $e^{-h}$, the expectancy of it being less than this high is $(1-e^{-h})$. The expectancy that none of *m independent* points is this high is

$$(1-e^{-h})^m = 1 - me^{-h} + \frac{m(m-1)e^{-h}}{2} - + \quad . \; . \; . \; .$$

Therefore the expectancy that at least one will be this high is

$$me^{-h} - \frac{m(m-1)e^{-h}}{2} + \frac{m(m-1)(m-2)e^{-3h}}{6} + \quad . \; . \; . \; . \; . \; .$$

Except for notation, this equation is identical with Walker's equation, which is developed in terms of an expectancy of one-half.

From here we will continue independently. Given, *m independent* points, we can choose any expectancy ratio *which satisfies our judgment of what is necessary to make physical reality of periods probable*. We can then solve for $h$ and if we find a peak of height $h$, we will assume it probably real. The expectancy ratio to be chosen as a criterion is a matter of judgment. One man will demand a higher ratio to satisfy him than will another. There can be no mathematical criterion which will set a definite value above which all will agree to the reality of the period. It will also vary with the physical probability of a period. For example, if we were working with rainfall data and knew of no periodicities in it nor in any other closely related meteorological data nor of any reason why we would expect a period, we would demand a higher ratio than if, perchance, a period had been established in temperature. Also, if a peak indicates a period of length simply related to some other plausibly connected phenomenon, we will not demand as high a ratio as otherwise.

Several high independent peaks are less likely to occur by accident than is one.

Let $p_1$ be the probability of any peak of height $h_1$. Suppose we have $z$ such peaks. The probability of their simultaneous occurrence is: $p_1 \quad p_2 \; . \; . \; . \; . \; p_z$.

This expression assumes that $z$ is very small with respect to $m$.

Walker states in paragraph six of the later paper: "If we have determined the values of, let us say, 20 amplitudes $C_k$ in the periodogram and have picked out the largest term, we must, if we wish to estimate the liklihood that the period is real, compare its amplitude, not with the probable value of a single random amplitudes but with the largest of 20 amplitudes produced by chance, and this will be 2.21 times as great."

The theory has been developed on the basis of $m$ *independent* points in a periodogram. The number of *computed* points is much larger than this. Let us illustrate by the rainfall paper cited above (*3*). A periodogram has been computed from 72 yearly values of data, with periodicities examined between 9 and 1⅙ years; 84 points have been computed for the periodogram. These are so close together that it is impossible for a peak to have a much greater $h$ than has been computed for it and certainly for no peak entirely to have escaped notice. Yet, pressing the application, the greater the number of such points one might compute, the higher he would expect to find peaks through mere accident.

There are as many entirely *independent* points within the periodogram as there are Fourier harmonics of the stretch of data between the limiting periods chosen for the periodogram. In this case, the eighth to twenty-third are included and we find 26 independent values instead of 84. This number, however, minimizes the independence too much, for if the highest point of the periodogram were to occur half way between two Fourier terms, there would be 90° phase divergence at the beginning and the end of the data between it and adjoining Fourier values. This would be sufficient to cause a material difference in height between the assumed highest peak and these terms. In other words, computing twice as many periodogram points as there are Fourier terms would cause us to expect higher peaks through accident. Thus we see that once again we are left to our judgment as to the criterion to be applied, this time in the number of terms to be considered as *independent*. The number is obviously less than the number of terms which should be computed for the periodogram, and greater than the number of Fourier terms of the stretch of data. Possibly some one may be able to develop a satisfactory criterion for everyone. I would estimate possibly 1½ times the Fouxier number as reasonable.

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## SOME OUTSTANDING TORNADOES

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(Abstract)[1]

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There is no reason to believe that in prehistoric days, nor during the early history of our country, tornadic storms were any less numerous or lacked the severity of those of to-day. As has been often pointed out, the greater completeness of the record in later years is simply a result of the increase in population affected by tornadoes. Thus an early list gives four times as many in the seventies as in the preceding 76 years, and contains very little data that would enable one to classify them as to magnitude. Complete statistics seem to be very meager prior to 1875.

With a view to having in compact form a record of the outstanding tornadoes that have occurred in the United States, the writer has compiled a list and brief description [2] of those which fall into any of the groups given below.

A. Storms with a death list of 50 or more,
B. With property loss of $500,000 or more,
C. With path more than 50 miles long,
D. With path 50 to 100 miles long,
E. With paths more than 100 miles long.

TABLE 1.—*Percentage distribution of the 4 groups of important tornadoes*

| Group | State having maximum number | Percentage in States: West of Mississippi | Percentage in States: East of Mississippi |
|---|---|---|---|
| A | Iowa, Texas, 10 per cent each | 53 | 47 |
| B | Kansas, 11 per cent | 44 | 56 |
| C | Iowa, 10 per cent | 47 | 53 |
| D | Missouri, 13 per cent | 56 | 44 |
| E | Alabama, 10 per cent | 31 | 69 |

The list contains 158 class one or outstanding tornadoes. In summarizing (Table 1) the results of the work, tornadoes that occurred in two States are given a weight of one-half in each and, likewise, when they occurred in three States a weight of one-third is charged to each.

According to the compilation, Iowa leads in important tornadoes with 9 per cent of the total number, the other States following in this order: Missouri, 8; Illinois, 7; Kansas, Tennessee, Alabama, Minnesota, and Wisconsin, each 6; Oklahoma and Indiana, 5 each. From the Plains States eastward, all States are represented except Maine, New Hampshire, Vermont, Rhode Island, Connecticut, New York, Virginia, and West Virginia—54 per cent occurred east of the Mississippi River.

In total number of deaths from these larger tornadoes Illinois leads by a wide margin, there being 631 deaths as a result of a single storm, a number greater than the total of all these class one tornadoes in any other State.

It was supposed that the 293-mile path of the "Mattoon" tornado of May 26, 1917, was the longest of record, but five earlier ones are now found exceeding 300 miles. Two of these had some long gaps, the information is very vague concerning two others, and it is not stated whether the fifth was continuously destructive. In our travels over the Illinois and Indiana portions of the tri-State tornado of March 18, 1925, we found absolutely no skipping in the 130 miles covered. The last-named storm exceeded all others in loss of life and value of property destroyed. The official report gave the number of deaths as 742, and the property loss at $16,500,000. There have been 8 tornadoes with a loss of life exceeding 100, and 2 with more than 135. In 15 storms the property loss has exceeded a million dollars, and in three of them it was ten million dollars or more.

Searching through some 700 records has brought out certain facts that are, perhaps, worthy of mention. A large number of tornadoes occur nearly every year, but many are of an incipient nature or do little damage. In some cases they have a tendency to form in groups, and to move in parallel paths in a northeasterly direction. A remarkable number continue for many miles over a straight path. The great tornado of March 1925, varied scarcely more than a mile or two from a straight line in 178 miles of its course. In a list of 384 tornadoes where the direction is stated, 78 per cent moved northeast, the others in some easterly direction. Of 452 tornadoes, 80 per cent were timed between noon and 6 p. m., and 15 per cent between 6 p. m. and midnight.

Kansas leads in number but the length of path probably averages less than farther east, and the greatest number of persons killed in a single tornado in that State was but 23. Tornadoes often occur when there are opposing northerly and southerly winds, with marked thermal difference.

It is questionable whether the observance of a funnel cloud should be made a requisite in defining a storm as a tornado.[1] In connection with the one in March, 1925, very few thought they saw a funnel cloud and these persons were not very definite. None was observed at St. Louis when that city was visited by a severe tornado on May 27, 1896. All through these old reports the statement recurs that two clouds came together, one from the northwest and one from the southwest. Professor Henry [2] says the character of the pendant funnel-shaped cloud varies with geographic position and the average hygrometric state of the air. It seems to the writer that if there is a long and narrow path, with an easterly movement of progression, it is safe to classify the storm as a tornado, especially if light débris aloft is thrown out to the left of the path.

[1] Paper read at meeting of American Meteorological Society, Kansas City, Dec. 28–29, 1925.
[2] These are on file in the library of the U. S. Weather Bureau, Washington, D. C.

[1] Dr. Humphreys stated that a funnel-shaped cloud was not always an accompaniment of a tornado though there was some confusion in the definition of the term. Mr. Reed mentioned a tornado in Iowa that had three branches, also one that had made a complete circle, with a radius of about 2 miles, in its path.
[2] Henry, Professor A. J., Monthly Weather Review, April, 1925.